Forecasting of Severe Thunderstorms using Upper Air data

Sonia Bhattacharya, Anustup Chakrabarty and Himadri Chakrabarty

Abstract— Severe local thunderstorm is the extreme weather convective phenomenon generated from cumulonimbus cloud. It has a devastating effect on human life. Correct forecasting is very crucial factor to save life and property. Here in this paper we have applied artificial neural network to achieve desired result. Multilayer perceptron has been applied on upper air data such as sunshine hour, pressure at freezing level, height at freezing level and cloud coverage (octa NH). MLP predicted correctly both 'squall' and 'no squall' storm days more than 90% with 12 hours leading time.

Index Terms— MLP, squall, cumulus cloud, sunshine hour, pressure at freezing level, height at freezing level, octa

1 INTRODUCTION

L hunderstorm is one of the most devastating type of mesoscale, convective weather phenomenon, generated from the cumulonimbus cloud. It occurs in different subtropical places of the world, (Ludlam, Over 40,000 1963). thunderstorms occur throughout the world each day[1] The strong wind which has the speed of at least 45 kilometers per hour with the duration of minimum 1 second is termed as squall [1]. It is accompanied with very strong surface wind, lightning, thunder, smart shower, and sometimes with hailstones [2]. A thunderstorm is associated with а very tall thundercloud mass (cumulonimbus cloud), which has a flat, dark base from which heavy rain and hail can fall.

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When not obscured by haze or other clouds, the top of a cumulonimbus is bright and tall, reaching up to an altitude of 10-16 km (lower in higher latitudes and higher in the tropics). Although a thunderstorm is a three-dimensional structure, it should be thought of as a constantly evolving process rather than an object. Each thunderstorm, or cluster of thunderstorms, is a self-contained system with organized regions of up drafts (upward moving air) and downdrafts (downward moving air). Their movement within the cloud and interaction with prevailing winds at various heights in the atmosphere form changing cloud features. The whole process is an example of convection, which acts to distribute energy more evenly in the atmosphere. Every thunderstorm cloud has a core region, a spreading anvil top, and an inflow-outflow region. The core is that part of the cloud where sustained strong up draughts of relatively warm and moist air condenses to produce rain, hail and/or snow (collectively known as precipitation) and associated downdraughts. Underneath the core we see a rain curtain, whilst above it the tallest part of the thunderstorm can be found. The dark flat cloud base that extends away from the core (usually to the west or north) is called the flanking line or rain-free base, along which air fuelling up draughts into the thunderstorm rises in successive cumulus towers. To produce a thunderstorm the atmosphere needs

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the right ingredients. These include moisture (sometimes indicated by low clouds or haziness in the morning and/or many cumulus clouds later), atmospheric instability to make the atmosphere more buoyant (often recognized by the presence of altocumulus castellanus, or turreted middle-level clouds, early in the day), and a lifting mechanism such as heating or the approach of a front or low pressure trough. The most likely severe weather days will have two essential factors in place. First, the atmosphere will be unstable enough to permit very strong up draughts to rise rapidly into colder air aloft. Second, the winds aloft are sufficiently strong to carry much of the leftover cloud matter well downwind and out of the way of warm air entering the system from below.

In this paper four parameters have been considered such as Sun Shine Hour (SSH), Pressure at Freezing level (FRZ), Height at Freezing level (FRZT) and Octa (NH). All these data were procured during the period of 33 years from 1990 to 2008 for the months of March-April-May (MAM). These three months are known as premonsoon season in north-east India. Most of the thunderstorms generally occur in this season. Neural network as an important branch of artificial intelligence has been applied to space weather forecasting such as forecasting of geomagnetic storm [3] and solar flair [4]. The objective is to develop a learning algorithm for a multilayer feedforward neural network, so that the network can be trained to capture the mapping implicit in the given set of input-output pattern pairs [5]. The training patterns are applied in some random order one by one, and weights are adjusted using the backpropagation learning law [5]. It is important to initialize the weight values properly before applying the learning law for a given training set [6][7]. Multi layer perceptron (MLP) and K-nn techniques have been applied by Chakrabarty et al., 2012 to predict squall-storms occurring in Kolkata using only two weather variables such as adiabatic lapse rate and moisture difference from surface level to five different geopotential heights of the atmosphere with around 12 hours lead time, resulting 91% accuracy in forecast. In the present work, nearly 100% accuracy both in the prediction of 'squall storm' and 'no squall-storm' have been obtained.

2 DATA

2.1 Data Collection

All the weather data were collected from India Meteorological Department, Govt. of India during the period of 33 years from 1969 to 2002 for the months of March-April-May. The data considered for analysis here are both for the days when squallstorms occurred and for some of the days when squall-storms did not occur. The numbers of 'squall-storm' days are 161 and 'no squall-storm' days are 327. The whole data-points have been divided into eleven sets, taking one set as testing set and remaining sets as training set. In each set squall and no-squall data has been arranged in 1:2 orders. Each set has been considered as test set and the remaining as training set for six different cases.

3 DATA DESCRIPTION

Four atmospheric parameters have been considered here for the analysis. These are moisture difference, dry adiabatic lapse rate and vertical wind shear. We have discussed about these three variables in the following sub-sections.

3.1 Sunshine Hour

Sunshine duration or sunshine hours is a climatologically indicator, measuring duration of sunshine in given period (usually, a day or a year) for a given location on Earth, typically expressed as an average of several years. It is a general indicator of cloudiness of a location, and thus differs from insolation, which measures the total energy delivered by sunlight over a given period. If the sun were to be above the horizon 50% of the time for a standard year consisting of 8760 hours, apparent maximal daytime duration would be 4380 hours. However, there are physical and astronomical effects which change that picture. Namely, atmospheric refraction allows the Sun to be still visible even when it physically sets below the horizon line. For that reason, average daytime (disregarding cloud effects) is longest in polar areas, where the apparent Sun spends the most time around the horizon. Places on the Arctic Circle have the longest total annual daytime of 4647 hours, while the North Pole receives 4575. Because of elliptic nature of the Earth's orbit, the Southern Hemisphere is not symmetrical: Antarctic Circle at 4530 hours receives 5 days less of sunshine than its antipodes. Since sunshine

duration depends chiefly on concentration of clouds and fog in the observed area, locations with dry, desert climate naturally correlate with high sunshine duration values, and, conversely, lowest values of sunshine duration are encountered in areas with wet, oceanic climate.

3.2 Pressure at Freezing Level (FRZ)

FRZ (freezing level) is the pressure level in the troposphere where the temperature is freezing. FRZ is located as the intersection between the 0 degree C isotherm and the temperature sounding. In the sounding below the FRZ level is 571 mb. You can verify this by following the 0 C isotherms from the bottom of the diagram up to the 571 mb pressure level. At this pressure level the sounding temperature will be intersected. To get our bearings, at 571 mb, the dew point is -10 C, temperature is 0 C and theoretical temperature an air parcel raised from PBL would be is 4 C. In a severe thunderstorm environment, a low FRZ level indicates hailstones will have more time to grow in the updraft and will have less time to melt as it falls to the surface. A FRZ level with a pressure level of 650 mb or closer to the surface in a severe weather situation generally will support large hailstones.

3.3 Height at Freezing Level (FRZHT)

The heights of the troposphere or tropopause up to which the cloud is deposited are to be noted down from the upper air radiosonde data. The height of the cloud from mean sea level is important to assume the tentative temperature of that height of the atmosphere. The height of the cloud is one of the factors for the formation of thundercloud. The initial stage for thunder cloud formation is the cumulus stage. The cumulus clouds surge upward to altitudes above 10 kilometers [8]. Cloud formation may occur either by free convection or by force convection. The continuous vertical growth of the cumulus conjestus cloud builds into a cumulonimbus cloud or thunderstorm cloud [8].

3.4 Cloud Coverage (Octa Nh)

The coverage of the cloud in the upper atmosphere indicates the atmospheric moisture content. This moisture content plays a vital role for the formation of the thundercloud. If the value of CAPE is suitable to form thunderstorm. The more the coverage of the cloud the more will be the moisture content of the upper air.

4 METHODOLOGY

A four-layered Multilayer Perceptron (MLP) network has been used in this study. It consists of an input layer, two hidden layer and an output layer. The input layer contains the sensory units of five input nodes where the first 4 nodes correspond to predictor weather variables, X1, X2, X3, X4 and the fifth one X5 is considered as the 'bias' term. The value of the fifth node is assumed as 1, irrespective of 'storm' or 'no storm' days. The input nodes X1, X2, X3 and X4 indicate Sunshine Hour, Pressure at Freezing Level, Height at Freezing Level and Octa. There is two hidden layer having several computation nodes and an output layer having two computation nodes.

4.1 Learning Phase

A learning law, called generalized delta rule or backpropagation law, is derived in this section [9][10]. Let (a_i, b_i) , l = 1, 2, ..., L be the set of training pattern pairs. It is not necessary to have all the training data set at one time, nor the training data set to be a finite set. The objective is to determine the weight update for each presentation of an input-output pair. Since the given data may be used several times during training, let us use the index *m* to indicate the presentation step for the training pair at step *m*. For training a multilayer feedforward neural network, we use the following estimate of the gradient decent along the error surface to determine the increment in the weight connecting the units *j* and *i*:

$$\Delta wij(m) = -\eta \frac{\partial E(m)}{\partial wij},\tag{1}$$

Where $\eta > 0$ is a learning rate parameter, which may also vary for each presentation of the training pair. The weight update is given by

 $w_{ij}(\mathbf{m}+1) = w_{ij}(\mathbf{m}) + \Delta w_{ij}(\mathbf{m}).$ ⁽²⁾

4.2 Connection weights

Connection weights (W's) are initialized to small random values in the range (-0.5 to 0.5). A threshold value is also assumed. The weight values are modified during back propagation of the learning of the model until the error is minimized. The modified weights are used to validate the testing datasets. The back propagation method basically used gradient descent technique for changing the weights. It is used to reduce the possibility of getting stuck in local optimal points or saddle points of the network.

4.3 Error

The error function is the mean square error, which is expressed by,

 $E = \sum_{j=1}^{2} \frac{(oj-ej)2}{2}$ (3)

The expected output (e_i) for every point in the

training set is known. For a particular observation, the actual output value for the j-th node in the output layer is o_j . This error is to be minimized during the training phase by the head propagation

during the training phase by the back propagation. Iteration is continued until the error is minimized around 0.005 to 0.001.

5 RESULTS

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6 CONCLUSION

It is observed from the above table that single hidden layer MLP (s) does not yield satisfactory result, accuracy is about 58.33%. Whereas using double hidden layer MLP we get an interesting result. For squall storm days it is about 94% and for no squall storm days it is more than 90%.

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